Optimal Transmission Ranges for Randomly Distributed Packet Radio Terminals

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Abstract—**In multihop packet radio networks with randomly distributed terminals, the optimal transmission radii to maximize the expected progress of packets in desired directions are determined with a variety of transmission protocols and network configurations. It is shown that the FM capture phenomenon with slotted ALOHA greatly improves the expected progress over the system without capture due to the more limited area of possibly interfering terminals around the receiver. The (mini)slotted nonpersistent carrier-sense-multiple-access (CSMA) only slightly outperforms ALOHA, unlike the single-hop case (where a large improvement is available), because of a large area of "hidden" terminals and the long vulnerable period generated by them. As an example of an inhomogeneous terminal distribution, the effect of a gap in an otherwise randomly distributed terminal population on the expected progress of packets crossing the gap is considered. In this case, the disadvantage of using a large transmission radius is demonstrated.**

I. INTRODUCTION

ONE of the key issues in providing efficient and cost-effective multihop packet radio networks is to find an adequate transmission power for each terminal in the network. The environment we have in mind is one in which communicating terminals are geographically distributed and possibly mobile and require multiaccess to a communication channel shared among themselves. It has been shown [7] that the spatial reuse of the channel obtained by reducing the transmission power to such a level that only a few neighbors are within the range gives rise to an improved throughput (the average rate of successful transmissions) for the network. However, since the purpose of transmitting packets in a multihop environment is to advance them towards their destinations, a more appropriate measure of performance is the expected one-hop progress of a packet in the desired direction [4], [7].

The optimal transmission power to maximize the expected progress involves the following tradeoff. (Here we assume every terminal uses the same power.) A short-range transmission is favorable in terms of successful transmission because of its low possibility of collision (the overlapping of packet transmission periods from multiple transmitters) at the receiver. A long-range transmission is favorable because 1) it moves a packet far ahead in one hop if successful, and 2) there is high probability of finding a candidate receiver in the desired direction. Roughly speaking, if we denote by N the average number of terminals within the transmission radius (N is clearly an increasing function of the radius), then the probability of successful transmission is proportional to $1/N$, whereas the progress is proportional to $\sqrt{N}$, and the contribution from the receiver's angular position is expressed as a monotonically increasing function of N from 0 to some asymptotic value. Thus, we see that there must exist an optimal value of N, which maximizes the obtainable expected progress.

This paper elaborates on these ideas with a variety of transmission protocols and network configurations. The protocols considered here include slotted ALOHA (with and without FM capture) [6] and nonpersistent carrier-sense-multiple-access (CSMA) [3], [8]. Terminals are randomly located in the plane according to a two-dimensional Poisson distribution with homogeneous or inhomogeneous density. Each section below begins with the description of the model used in that section, followed by the formulation of the optimization problem. The optimal transmission range is found, and the performance is compared to other models. The results are summarized in the concluding section.

II. OPTIMAL TRANSMISSION RADII FOR SLOTTED ALOHA

This section is concerned with the optimal transmission radii for randomly distributed terminals using slotted ALOHA as the transmission protocol. The same problem was considered by Kleinrock and Silvester [4], [7] who provided the "magic number" 6 as the optimal number of terminals to be covered by one transmission. However, there appears to be an inconsistency in their treatment. (In evaluating the probability of successful reception [7, eq. (6.7)], the number of terminals around the receiver is confused with that around the transmitter. As a matter of fact, the resultant optimal p, $p^* = 1/N$, could be greater than 1 [inconsistent with slotted ALOHA] for a very small transmission radius.) Therefore, we reconsider their problem and show a different magic number nearly equal to 8. The present section also serves to provide the most basic model among those considered in this paper.

We consider the progress that a given packet makes in the direction towards its final destination for a single (arbitrary) slot only and do not discuss its behavior along the entire path. The basic assumptions and associated parameters used in this section are as follows.

Transmission protocol: slotted ALOHA. The slot length in time is equal to the transmission time of a packet. (All packets are assumed to be of the same length.) The propagation time is ignored (or considered to be included in the slot). We do not take into account the acknowledgment traffic. It is assumed that the successful reception of a packet is immediately made known to the transmitter (e.g., by using a different (free) channel of wide bandwidth).

Transmission probability: p. All terminals are supposed to

Paper approved by the Editor for Computer Communications of the IEEE Communications Society for publication without oral presentation. Manuscript received January 6, 1983; revised May 2, 1983. This work was supported by the Defense Advanced Research Projects Agency of the Department of Defense under Contract MDA 903-77-C-0272.

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0090-6778/84/0300-0246$01.00 © 1984 IEEE

have packets at all times (heavy-traffic assumption). For every slot, each terminal transmits a packet with probability p (and does not with probability $1 - p$), where $0 < p \leqslant 1$.

Transmission radius: R. All terminals use the same transmission radius. This means that terminals within a circle of radius R centered at the transmitter hear the transmission, whereas others do not hear it at all. More than one transmission within a distance R of the receiver in the same slot bring about the collision of all packets at that receiver.

Spatial distribution of terminals: two-dimensional Poisson distribution with the average number of terminals per unit area λ. We assume that a new sample of the spatial distribution is given for every slot.

Distribution of the sources and destinations of packets: two-dimensional isotropic, i.e., uniform over the plane. For every slot, the direction of the final destination for a packet in each terminal is assumed to be uniformly distributed in angle.

Routing strategy: most forward within R (MFR). Each terminal is assumed to know the position of those terminals within a distance R. Given a packet and its final destination, a terminal transmits to the terminal most forward (among those whose positions it knows) in the direction of the final destination. If no terminals are in the forward direction, it transmits to the least backward terminal, if any. (A terminal cannot transmit to itself.) In case there are no terminals in the circle of radius R at all, it does not transmit in that slot. (Note that MFR may not be minimizing the remaining distance to be traveled to the destination; MFR is myopic routing.)

$N \triangleq \lambda\pi R^2$: the average number of terminals within a radius R, and also a measure of connectivity of the network.

In this environment, we have the following two measures of performance.

$S(p, N) \triangleq$ the one-hop throughput, defined as the average number of successful transmissions per slot from a terminal.

$Z(p, N) \triangleq$ the expected progress of a packet in the direction of its final destination per slot from a terminal. The progress x is attained when x is the distance between the transmitter and the receiver projected onto a line drawn towards the final destination and the transmission to that receiver is successful.

Note that $Z(p, N)$ has the dimension of length (e.g., miles). Therefore, $Z(p, N)\sqrt{\lambda}$ may conveniently be used as a dimensionless measure of the expected progress in the number of hopped-over terminals since $1/(2\sqrt{\lambda})$ is the average distance between two nearest terminals. (See (37) below.) We employ $Z(p, N)$ as the objective function for our optimization problem in accordance with the routing strategy MFR. A point in the (p, N) plane which maximizes $Z(p, N)$ is sought. However, the value of $S(p, N)$ at this optimal point is also interesting. It will turn out that the same $p = p^*(N)$ maximizes both $S(p, N)$ and $Z(p, N)$.

In order to evaluate $S(p, N)$, we first note that e^{-N} is the probability of there being no terminals within a distance R of the transmitter. In such a case, no transmission can occur. Under the condition that there is at least one candidate rereceiver within R, let A_i be the event that there are i other terminals (excluding the transmitter P and receiver Q) within a distance R of Q. (See Fig. 1.) Thanks to the memoryless property of the Poisson distribution, the distribution of the number of other terminals within R does not depend on the existence of P and Q. Thus, we have

$$\text{Prob}\,[A_i] = \frac{N^i}{i!}e^{-N} \qquad i = 0, 1, 2, \cdots. \tag{1}$$

The transmission from P to Q is successful (let this event be

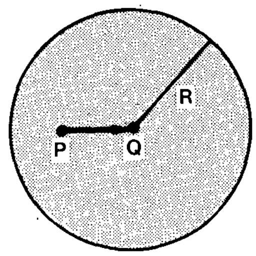

Fig. 1. The area of interference with the transmission $P \to Q$.

denoted by $P \to Q$) if none of the terminals within a distance R of Q transmit (including Q itself). Thus,

$$\text{Prob}\,[P \to Q \mid A_i] = (1-p)^{i+1}. \tag{2}$$

It follows that

$$\begin{aligned} S(p, N) &= \text{Prob}\,[\text{there is at least one terminal within } R] \\ &\quad \cdot \text{Prob}\,[P \text{ transmits}] \cdot \text{Prob}\,[P \to Q] \\ &= (1-e^{-N})p \sum_{i=0}^{\infty} \text{Prob}\,[P \to Q \mid A_i]\,\text{Prob}\,[A_i] \\ &= p(1-p)e^{-pN}(1-e^{-N}) \qquad (3) \\ &\cong p(1-p)N \quad \text{as } N \to 0, \\ &\cong p(1-p)e^{-pN} \quad \text{as } N \to \infty. \end{aligned}$$

Given N, $S(p, N)$ is maximized by

$$\begin{aligned} p = p^*(N) &\triangleq \frac{2}{N + 2 + \sqrt{N^2+4}} \qquad (4) \\ &\cong 1/2 \quad \text{as } N \to 0, \\ &\cong 1/N \quad \text{as } N \to \infty. \end{aligned}$$

The maximum value itself is given by

$$\begin{aligned} S(p^*(N), N) &= \frac{1-e^{-N}}{2+\sqrt{N^2+4}} \exp\left(-\frac{2N}{N+2+\sqrt{N^2+4}}\right) \\ &\cong (1/4)N \quad \text{as } N \to 0, \qquad (5) \\ &\cong 1/(Ne) \quad \text{as } N \to \infty. \end{aligned}$$

In the case $N \to 0$, at most only pairs of terminals can hear each other. So, they transmit with probability 1/2 (the well-known optimal p for two terminals). The optimized results for large N are the same is in [4] and [7], in which it is stated that they correspond to setting the average traffic load to be equal to one packet per slot within the transmission range. The optimized throughput of N terminals for large N, $1/e$, also conforms to Roberts' result [6] for a single-hop system of an infinite number of terminals.

We now proceed to find $Z(p, N)$. According to the MFR routing, $\tilde{z}$, the progress of a packet per transmission, is not greater than x if there are no terminals in the area A in Fig. 2.

$$\text{Prob}\,[\tilde{z} \leqslant x] = e^{-(N/\pi)q(x/R)} \tag{6}$$

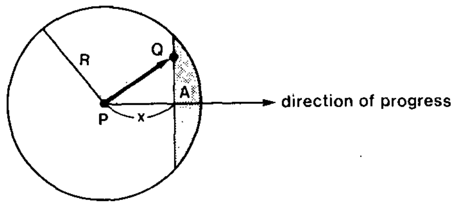

Fig. 2. The position of the receiver Q.

where

$$q(t) \triangleq \cos^{-1}(t) - t\sqrt{1-t^2}. \tag{7}$$

Note that $q(t)$ is the area of A in Fig. 2 when $R = 1$ (unit circle) and $x = t$. Therefore, we have

$$\begin{aligned} Z(p, N) &= \text{Prob}\,[P \text{ transmits}] \cdot \text{Prob}\,[P \to Q] \\ &\quad \cdot E\,[\text{progress of a packet}] \\ &= p(1-p)e^{-pN} \int_{-R}^{R} x \cdot \text{Prob}\,[x < \tilde{z} \leqslant x + dx] \\ &= p(1-p)e^{-pN} \sqrt{\frac{N}{\lambda\pi}} \Bigg[1 + e^{-N} \\ &\quad - \int_{-1}^{1} e^{-(N/\pi)q(t)}\, dt\Bigg]. \end{aligned} \tag{8}$$

Thus, given N, $Z(p, N)$ is also maximized by $p = p^*(N)$ given by (4), and the normalized maximum is given by

$$\begin{aligned} Z(p^*(N), N)\sqrt{\lambda} &= \frac{1}{2 + \sqrt{N^2+4}} \exp\left(-\frac{2N}{N + 2 + \sqrt{N^2+4}}\right) \\ &\quad \cdot \sqrt{\frac{N}{\pi}} \left[1 + e^{-N} - \int_{-1}^{1} e^{-(N/\pi)q(t)}\, dt\right] \\ &\cong \frac{16}{45}\left(\frac{N}{\pi}\right)^{5/2} \quad \text{as } N \to 0, \\ &\cong \frac{1}{e\sqrt{\pi N}} \quad \text{as } N \to \infty. \end{aligned} \tag{9}$$

The functions $p^*(N)$, $S(p^*(N), N)$, and $Z(p^*(N), N)\sqrt{\lambda}$ as given by (4), (5), and (9), respectively, are plotted in Fig. 3. $Z(p^*(N), N)\sqrt{\lambda}$ has its maximum value at

$$N = N^* = 7.72.$$

Thus, we propose a new magic number, 8, as the optimal number of terminals to be covered in the transmission range. In terms of transmission radius, we have

$$R^* = 3.14(1/(2\sqrt{\lambda})).$$

The associated optimal values are

$$p^* \triangleq p^*(N^*) = 0.113$$

$$S^* \triangleq S(p^*, N^*) = 0.0419$$

$$Z^*\sqrt{\lambda} \triangleq Z(p^*, N^*)\sqrt{\lambda} = 0.0431.$$

Therefore, the sketch of optimal transmission is described as follows. Each terminal transmits a packet in every ninth slot on the average ($1/p^* = 8.85$). The probability of success of such a transmission is $S^*/p^* = 0.37$, as slotted ALOHA predicts. It uses a transmission radius to span just about three (3.14) nearest neighbors in linear distance. Then, the expected progress of the packet is $Z^*/p^* = 0.76(1/(2\sqrt{\lambda})) \cong (2/3)(R^*/e)$. Here the factor $1/e$ accounts for the probability of successful transmission, and $(2/3)\ R^*$ represents the effective distance that a packet is advanced by a successful transmission with radius R^*.

III. Optimal Transmission Radii for ALOHA with Capture

The analysis of the preceding section is here extended to the case of a slotted ALOHA system with FM capture. The observation that the capture phenomenon increases the throughput for a single receiver has been investigated by Roberts [6] and Abramson [1]. Fratta and Sant [2] have shown how capture affects the throughput behavior of an ALOHA network which has multiple transmitters and receivers. They did not use the notion of the transmission radius as we have done in Section II. Their work will be the basis of Section VI. In this section, we consider the optimization problem of the expected progress of packets through the MFR (most forward within the transmission radius R) routing in a capture environment. Similar work has been done by Nelson [5] using a different routing strategy. (Specifically, in his routing, one of the [say] k terminals within a half circle [in the forward direction] of radius R is picked as a receiver with probability $1/k$. As a result, his optimized expected progress is somewhat smaller than ours. For example [using the notation defined below], in the case of perfect capture, he gives $Z^*/R^* = 0.0346$, while we give $Z^*/R^* = 0.0393$.)

The basic assumptions and parameters for the model we study here are the same as in Section II, except for the conditions for successful transmission. They include the slotted ALOHA transmission protocol, transmission probability p, transmission radius R, Poisson distribution of terminals with parameter λ, MFR routing, isotropic distribution of source-destination pairs, and $N \triangleq \lambda\pi R^2$.

The concept of FM capture used in this section and Section VI is the same as in the papers cited above, that is, a receiver will correctly receive a packet from a transmitter which is located at a distance r of the receiver if none of the terminals within a distance αr of the receiver transmit simultaneously. The capture parameter α is related to the capture ratio CR in decibels via $\text{CR} = 20 \log_{10}\alpha$, $1 \leqslant \alpha < \infty$. The case $\alpha = 1$ is called perfect capture, whereas the case $\alpha \to \infty$ corresponds to the system without capture (i.e., the case considered in Section II).

Under these circumstances, we evaluate the throughput $S(p, N; \alpha)$ and the expected progress $Z(p, N; \alpha)$. We employ $Z(p, N; \alpha)$ as the objective function of our optimization problem with respect to p and N. [Now the optimum $p^*(N; \alpha)$ for $S(p, N; \alpha)$ is different from that for $Z(p, N; \alpha)$.]

First, we state the conditions for successful transmission of a packet. Since all terminals are using the same transmission radius R, the transmission from the transmitter P to the receiver Q, under the condition that they are a distance r apart, is successful if no other terminals within a distance

$$r' \triangleq \min\,[\alpha r, R] \tag{10}$$

of Q (including Q itself) transmit at the same time [5], [6]. Fig. 4 shows the area of potential interfering terminals for the transmission from P to Q. Thus, unconditioning on the num-

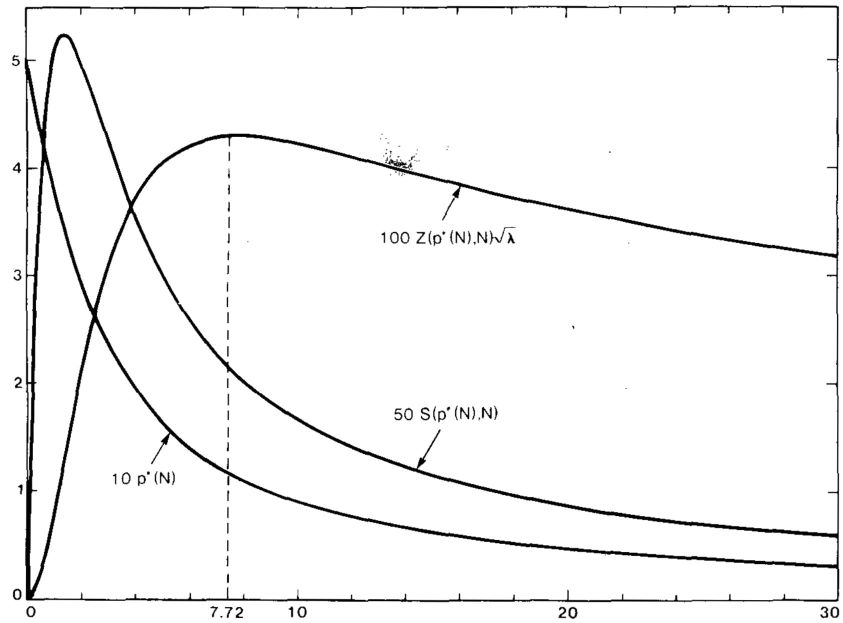

Fig. 3. The optimal transmission for slotted ALOHA networks without capture. (In this and some other figures, for conciseness we have plotted scaled values against a single vertical axis.)

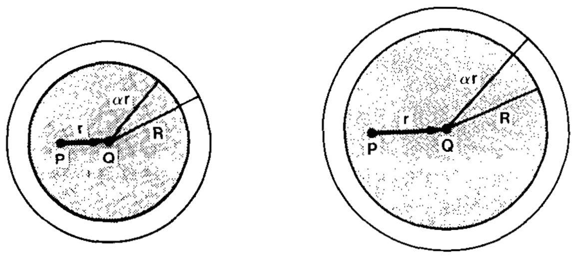

Fig. 4. The area of terminals possibly interfering with the transmission $P \to Q$. (a) $\alpha r < R$. (b) $\alpha r > R$.

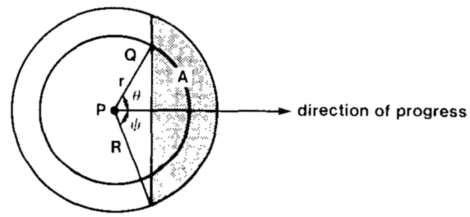

Fig. 5. The position of the receiver Q with respect to the position of the transmitter P.

ber of terminals in the area as in Section II, we have

$$\text{Prob}\,[P \to Q \mid \tilde{r} = r] = (1-p)e^{-\lambda p \pi r'^2} \tag{11}$$

where $P \to Q$ represents the event that the transmission from P to Q is successful, and $\tilde{r}$ is the distance between P and Q.

Second, we need the expression for the distribution of the positions of the receiver with respect to the transmitter. Let $(\tilde{r}, \tilde{\theta})$ be the polar coordinates of the position of the receiver Q, where the origin of the coordinates is at the position of the transmitter P, and $\tilde{\theta}$ is measured from the direction in which a packet at P is destined to proceed. See Fig. 5 for the configuration. Let A be the shaded area. Due to the MFR routing, the receiver is located at (r, θ) if and only if there are no terminals in A and there is a terminal at (r, θ). Therefore,

$$\begin{aligned}
&\text{Prob}\,[r < \tilde{r} \leqslant r + dr, \theta < \tilde{\theta} \leqslant \theta + d\theta] \\
&= e^{-\lambda R^2(\psi - \sin\psi\cos\psi)} 2\lambda r\, dr\, d\theta \\
&= 2\lambda r e^{-(N/\pi) q((r/R)\cos\theta)}\, dr\, d\theta \\
&\qquad 0 \leqslant r \leqslant R,\ 0 \leqslant \theta \leqslant \pi
\end{aligned} \tag{12}$$

where we have used the relation $r \cos\theta = R \cos\psi$ and the definition of $q(t)$ given in (7). Note that this is an expression for an event similar to the event defined for (6).

Thus, we have the throughput [similar to (3)]

$$\begin{aligned}
S(p, N; \alpha) &= p \int_0^R \int_0^\pi (1-p)e^{-\lambda\pi p r'^2} \\
&\quad \cdot \text{Prob}\,[r < \tilde{r} \leqslant r + dr, \theta < \tilde{\theta} \leqslant \theta + d\theta] \\
&= \frac{2}{\pi} pN(1-p) \int_0^1 t e^{-pNt'^2}\, dt \\
&\quad \cdot \int_0^\pi e^{-(N/\pi) q(t\cos\theta)}\, d\theta
\end{aligned} \tag{13}$$

where

$$t' = \min\,[\alpha t, 1]. \tag{14}$$

We see that (13) reduces to (3) when $\alpha \to \infty$. Also, we have

$$S(p, N; \alpha) \cong p(1-p)N$$

for $N \ll 1$ and a moderate value of α, which is again the same as before.

The expected progress can be obtained similarly as

$$Z(p, N; \alpha) = p \int_0^R \int_0^\pi (1-p) e^{-\lambda \pi p r'^2} r \cos\theta$$
$$\cdot \text{Prob}\,[r < \tilde{r} \leqslant r + dr, \theta < \tilde{\theta} \leqslant \theta + d\theta]$$
$$= \frac{2}{\pi} pN(1-p) \sqrt{\frac{N}{\lambda\pi}} \int_0^1 t^2 e^{-pNt'^2}\, dt$$
$$\cdot \int_0^\pi \cos\theta e^{-(N/\pi) q(t\cos\theta)}\, d\theta. \tag{15}$$

The maximum of $Z(p, N; \alpha)\sqrt{\lambda}$ is sought in the (p, N) plane for given α. Let $p^*(N; \alpha)$ be the p that achieves this maximum for given N and α. The optimal point for $\alpha = 1$ (perfect capture) is found as follows:

$N^* = 7.1$ or $R^* = 3.0(1/(2\sqrt{\lambda}))$

$p^* \triangleq p^*(N^*; 1) = 0.17$

$S^* \triangleq S(p^*, N^*; 1) = 0.068$

$Z^*\sqrt{\lambda} \triangleq Z(p^*, N^*; 1)\sqrt{\lambda} = 0.059.$

The expected progress is about 36 percent better than the system without capture. The optimal transmission is now sketched as follows. Each terminal transmits a packet in every sixth slot on the average ($1/p^* = 5.88$). The probability of successful transmission is $S^*/p^* = 0.4 > 1/e$. The transmission radius used is three times the average distance between the two nearest neighbors. Then, the expected progress of a packet per transmission is $Z^*/p^* = 0.69(1/(2\sqrt{\lambda}))$.

Fig. 6 displays the optimal values of parameters N and p and resulting S^* and $Z^*\sqrt{\lambda}$ for various values of the capture parameter α. From [6], good FM corresponds to CR = 1.5, while moderate FM corresponds to CR = 3.0 and poor FM corresponds to CR = 6.0. As we noted earlier, the results for $\alpha = \infty$ (no capture) coincide with those in Section II. We first notice that $Z^*\sqrt{\lambda}$ with some capture is always greater than that without capture. Thus, a conclusion here is that the FM capture always helps the progress of packets. The reason for this is that we limit the area of interfering terminals within $\min[\alpha r, R]$, which is always no greater than R for the case without capture. This implies a smaller number of interfering terminals, thus giving higher throughput and greater expected progress. It is also interesting that as α increases, N^* first decreases and then increases to reach its final value. This might be explained as follows. For small α, the limitation of a conflicting area by αr is more effective than that by R, so N^* decreases with more conflict as α increases. On the other hand, for large α, the limitation by R is dominant; so N^* approaches the value without capture.

IV. Optimal Transmission Radii for CSMA

In a single-hop network, another (great) improvement over ALOHA is made possible by CSMA. With this protocol, each terminal utilizes the information about channel status (busy or idle) obtained by listening to the channel. However, the existence of some terminals which are not in line of sight of others causes degradation in performance; this is called the hidden-terminal effect [8], [9]. If we use the CSMA protocol in a multihop network, we expect a similar effect because the hearing range of the receiver is more or less different from the listening range of the transmitter. The purpose of this section is to estimate the effect of hidden terminals associated with CSMA, with the same terminal distribution and with the same packet routing strategy as in the preceding sections. The basic assumptions and parameters carried over from Section II include the Poisson distribution of terminals with parameter λ, transmission radius R, MFR (most forward within R) routing, isotropic distribution of source-destination pairs, and $N \triangleq \lambda\pi R^2$.

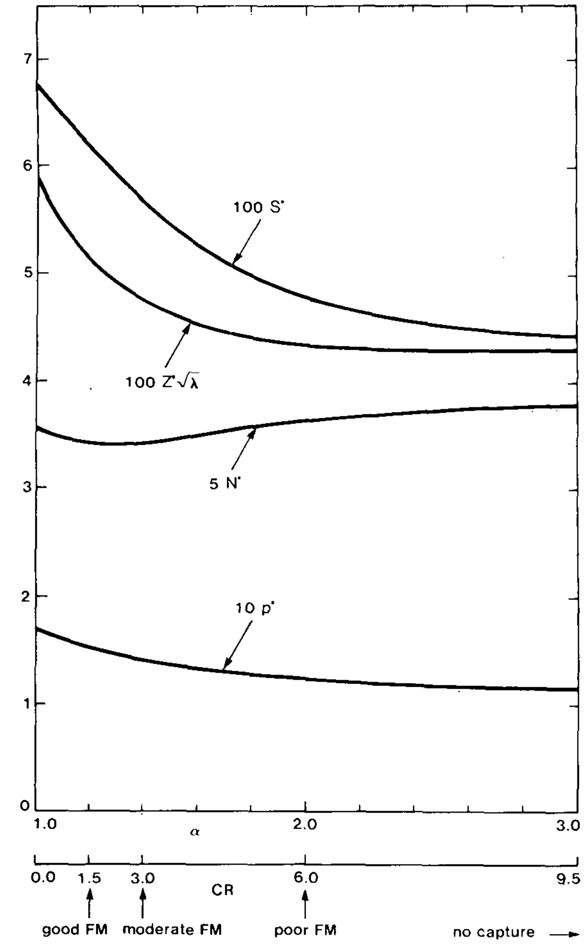

Fig. 6. The optimal transmissions for slotted ALOHA with capture.

We now explain the protocol of slotted nonpersistent CSMA. The constant packet transmission time is chosen as the unit of time, and the length of a (mini)slot, denoted by a, accounts for the signal propagation delay. In the derivation below, $\tau \triangleq 1/a$ is assumed to be an integer. (Propagation delay a is used to imply a time interval long enough for all the terminals in the transmission range to recognize the events that occurred time a before.) See Fig. 7 for the illustration of the channel activity heard at the receiver. We assume that all the terminals within a distance R of the transmitter recognize the transmission in one slot and that they hear the transmission one slot more after the completion of transmission. Assuming that every terminal is ready to transmit at all times, the nonpersistent protocol is described as follows. In every slot, each terminal listens to the channel with probability p (and does not with probability $1 - p$). That is, the channel-sensing behavior in a sequence of slots (except during the transmission) at each terminal constitutes independent Bernoulli trials. The parameter p is the sensing rate per slot. If the channel is sensed idle, it begins transmission in the same slot with probability 1. If the channel is sensed busy, it suppresses the transmission, and stops sensing the channel until the end of the current transmission. When the channel becomes idle, the above sensing procedure is repeated.

It is clear that the events whether an actual transmission occurs or not as a result of channel sensing in a sequence of slots

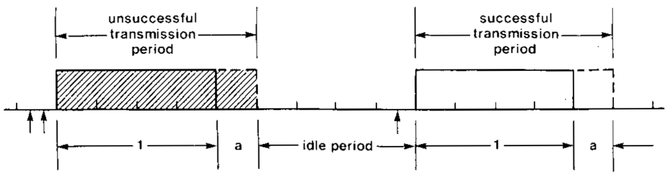

Fig. 7. Slotted nonpersistent CSMA. Transmission and idle periods.

at each terminal are no longer independent Bernoulli trials. However, we introduce the assumption that they are. That is, for every slot (except during the transmission), each terminal transmits a packet with probability p' (and does not with probability $1 - p'$). A similar assumption was used in [9], and the validity of results obtained was claimed by comparing the throughput values against simulation. The parameter p' is the transmission rate per slot. We leave the determination of p' in terms of p to the Appendix since we formulate our optimization problem with only p'. Under these conditions, we will evaluate the throughput of transmission $S(p', N; a)$, and the expected progress $Z(p', N; a)$. We employ $Z(p', N; a)$ as the objective function of our optimization problem with respect to p' and N. [Again $S(p', N; a)$ and $Z(p', N; a)$ are not optimized by the same $p'(N)$.]

A particular transmission is successful when no other terminals within a distance R of the receiver transmit during the transmission period $1 + a$. Let us consider the conditions for the successful transmission from the transmitter P to receiver Q, referring to Fig. 8. The shaded area A and B shows the area of terminals whose transmission may collide with the transmission from P to Q at Q. Since the terminals in area A recognize the transmission in one slot, a collision will be avoided if they do not begin transmission in the same slot. On the other hand, since the transmissions from the terminals in area B occur independently, it is sufficient that they keep silent throughout the entire vulnerable period of length $2 + a$ or $2\tau + 1$ slots shown in Fig. 8(b) (the first τ slots are included so as to prevent any interference with the ongoing transmissions and the second $\tau + 1$ slots are included not to be interfered with newly started transmissions). (Two packets whose transmissions start with τ slots apart may or may not be received successfully; however, we exclude such a case to pessimistically evaluate the probability of success.) Therefore, if $\tilde{r}$ denotes the distance between P and Q, and $P \to Q$ denotes the successful transmission from P to Q, then we have

$$
\begin{aligned}
&\operatorname{Prob}[P \to Q \mid \tilde{r} = r] \\
&\quad = \operatorname{Prob}[Q \text{ does not start transmission in the same slot}] \\
&\qquad \cdot \operatorname{Prob}[\text{no transmission from } A \text{ during a slot} \mid \tilde{r} = r] \\
&\qquad \cdot \operatorname{Prob}[\text{no transmission from } B \text{ during } 2\tau + 1 \\
&\qquad\quad \text{slots} \mid \tilde{r} = r].
\end{aligned}
$$

Since the area of A is $2R^2 q(r/2R)$ and the area of B is $\pi R^2 - 2R^2 q(r/2R)$, we get

$$
\begin{aligned}
&\operatorname{Prob}[P \to Q \mid \tilde{r} = r] \\
&\quad = (1 - p') e^{-p'\lambda \cdot 2R^2 q(r/2R)} \\
&\qquad \cdot e^{-(2\tau+1)p'\lambda \cdot [\pi R^2 - 2R^2 q(r/2R)]} \\
&\quad = (1 - p') e^{-p'N\{1 + 2\tau[1 - (2/\pi) q(r/2R)]\}}
\end{aligned} \tag{16}
$$

where $q(t)$ is defined in (7). An assumption involved here is that an independent sample of terminal distributions is given afresh for every slot throughout the vulnerable period from B.

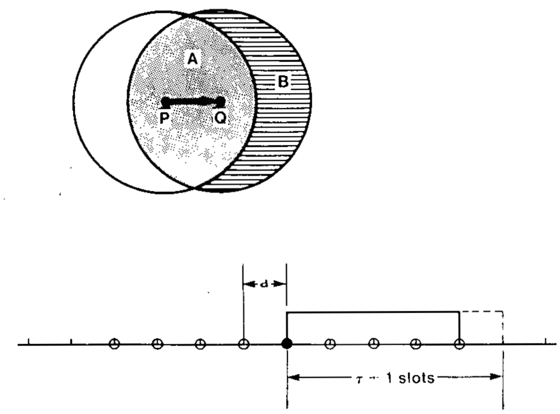

Fig. 8. The period for the transmission P to Q vulnerable to the transmissions from areas A and B. (a) Configuration. (b) Time line (•: vulnerable points to A and B; o: vulnerable points to B).

Based on this assumption we have evaluated the probability of success in each slot independently.

Since the assumptions about routing are the same as in Section III, the distribution of the position $(\tilde{r}, \tilde{\theta})$ of the receiver Q with respect to the transmitter P is given by (12). It follows that the one-hop throughput is given by

$$
\begin{aligned}
S(p', N; a) &= \frac{p'}{a} \int_0^R \int_0^\pi \operatorname{Prob}[P \to Q \mid \tilde{r} = r] \\
&\qquad \cdot \operatorname{Prob}[r < \tilde{r} \leqslant r + dr, \theta \leqslant \tilde{\theta} < \theta + d\theta] \\
&= \frac{2}{\pi} p' \tau N (1 - p') e^{-p'(2\tau+1)N} \\
&\qquad \cdot \int_0^1 t e^{(4p'\tau N/\pi) q(t/2)} \, dt \\
&\qquad \cdot \int_0^\pi e^{-(N/\pi) q(t \cos\theta)} \, d\theta
\end{aligned} \tag{17}
$$

and similarly the expected progress is given by

$$
\begin{aligned}
Z(p', N; a) &= \frac{2}{\pi} p' \tau N (1 - p') \sqrt{\frac{N}{\lambda\pi}} \, e^{-p'(2\tau+1)N} \\
&\qquad \cdot \int_0^1 t^2 e^{(4p'\tau N/\pi) q(t/2)} \, dt \\
&\qquad \cdot \int_0^\pi e^{-(N/\pi) q(t \cos\theta)} \, d\theta.
\end{aligned} \tag{18}
$$

(We note that the above $S(p', N; a)$ and $Z(p', N; a)$ are not the long-time average values because we have not taken into account the channel activity cycles (idle and busy) whose duration is variable. Thus, (17) and (18) may be viewed as giving the instantaneous values at transmission start times; note that S and Z in slotted ALOHA cases are overall means and instantaneous values at the same time. Thus, the comparison between CSMA and ALOHA is meaningful.)

The maxima of the function $Z(p', N; a)\sqrt{\lambda}$ are determined numerically in the (p', N) plane for various values of a. The optimal point for $a = 0$ (zero propagation delay) is found as

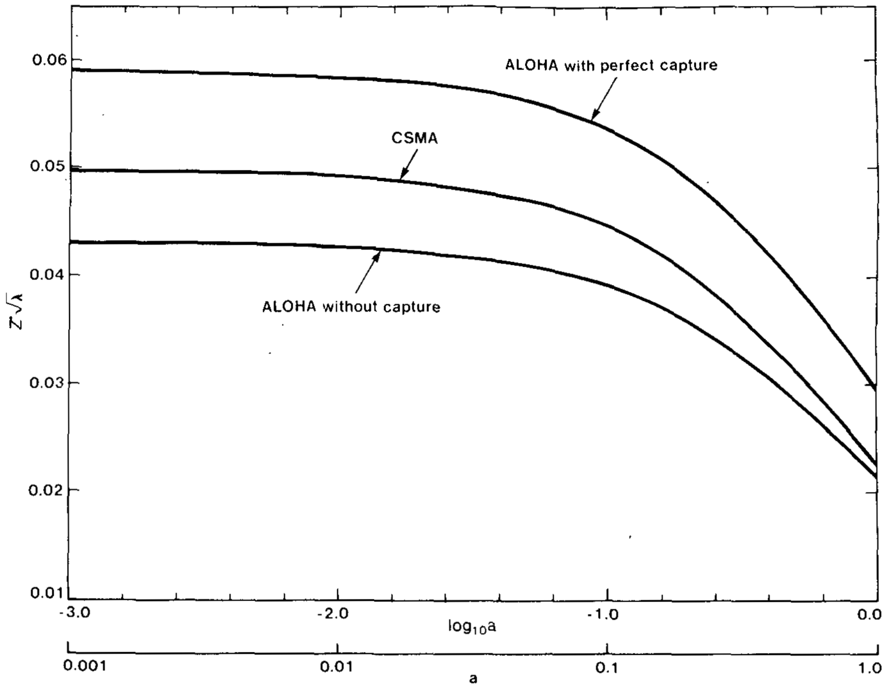

Fig. 9. Comparison of the optimized expected progress among ALOHA with and without capture, and CSMA networks.

follows:

$$N^* = 5.3 \quad \text{or} \quad R^* = 2.6(1/(2\sqrt{\lambda}))$$

$$\lim_{a\to 0} \frac{p^*}{a} = \lim_{\tau\to\infty} \tau p^* = 0.20$$

$$S^* \triangleq S(p^*, N^*; 0) = 0.077$$

$$Z^*\sqrt{\lambda} \triangleq Z(p^*, N^*; 0)\sqrt{\lambda} = 0.050.$$

Therefore, the optimized expected progress is only about 16 percent ($\doteq(0.050 - 0.0431) \times 100/0.0431$) better than ALOHA system without capture.

This small improvement in performance, unlike the single-hop case, appears due to the large area of hidden terminals (about half of the hearable range for $N = N^*$) and the long period (twice as long as the packet transmission time) vulnerable to their transmission.

In Fig. 9, the optimized expected progress with CSMA is plotted for various values of a, together with those for ALOHA systems without capture and with perfect capture. (For proper comparison, the optimized expected progress with slotted ALOHA should be divided by $1 + a$ to include the propagation time in a slot.) It is seen that the performance of CSMA lies between ALOHA without capture and ALOHA with perfect capture. With reference to Fig. 6, CSMA's performance turns out comparable to that of ALOHA with capture ratio about 1.5 dB which corresponds to good FM. The degradation of the expected progress with increasing a is due to the longer vulnerable period.

V. Optimal Transmission Radii in an Inhomogeneous Density of Terminals

So far we have considered only the Poisson distribution of terminals with the same spatial density everywhere. However, it is of importance in our multihop packet radio studies to extend the analysis to inhomogeneous structures. For example, how should the transmission power be controlled as one passes from a region of low density terminals to higher density terminals and then back out again to lower density terminals; this corresponds to a kind of geographical bottleneck. Another configuration is what we call the "dumbbell" configuration in which we have high density regions (say, two cities) connected together with an extremely low density region (say, a desert). Here one inquires whether the low density region helps the transmission or not. These are some of the motivations for our study of inhomogeneous configurations of packet radio terminals.

Specifically, the configuration of terminals we consider in this section is a vacant strip of width b in an otherwise Poisson-distributed terminal population with uniform average density λ. Taking the x-axis perpendicular to the gap length, the average density of terminals at x is given by

$$g(x) \triangleq \begin{cases} 0 & 0 \leqslant x \leqslant b \\ \lambda & \text{elsewhere.} \end{cases} \tag{19}$$

We introduce the "intensity" of the gap by

$$\beta \triangleq \lambda b^2 \tag{20}$$

which is the average number of terminals that would be in the gap of length b if it were not for the gap. The dimensionless quantity β will be used as a characteristic parameter below.

In the following we evaluate the expected progress of a packet residing at the terminal P on the left bank ($x = 0$) and destined to cross the gap. See Fig. 10 for the configuration. We assume slotted ALOHA protocol and the transmission radius R ($>b$) for all terminals. For simplicity, we do not optimize the transmission probability p but will use the value $p = 0.113$ which has been found optimal for the case of homogeneous Poisson distribution (see Section II). We recognize the terminals being with a distance R of the receiver as those which may cause conflict with our transmission. Then, our usual procedure yields the probability of successful trans-

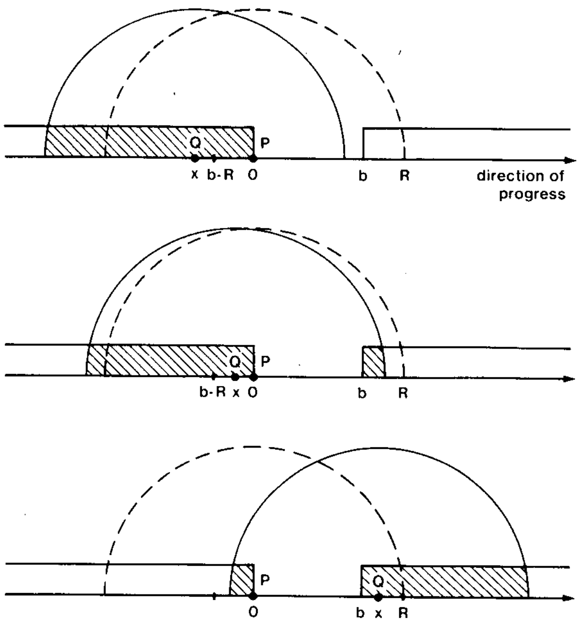

Fig. 10. Three cases of the position of the receiver Q. P: the transmitter, ////: the area of possibly interfering terminals. (a) $x \leqslant b - R$. (b) $b - R \leqslant x \leqslant 0$. (c) $b \leqslant x \leqslant R$.

mission to the receiver at x as

$$p(1-p)e^{-pn(x;R)} \tag{21}$$

where $n(x; R)$ is the average number of terminals within a circle of radius R around the receiver at x, given by

$$n(x;R) = \begin{cases} \lambda R^2 \left[\pi - q\left(-\frac{x}{R}\right)\right] & -R \leqslant x \leqslant b-R \\ \lambda R^2 \left[q\left(\frac{b-x}{R}\right) + \pi - q\left(-\frac{x}{R}\right)\right] & b-R \leqslant x \leqslant 0, \quad b \leqslant x \leqslant R \end{cases} \tag{22}$$

where $q(t)$ is defined in (7). The probability distribution function $F(x; R)$ of the position of the receiver is given by

$$F(x;R) \triangleq \text{Prob}\,[\text{no terminal in } (x, R) \mid R] = \begin{cases} \exp\left\{-\lambda R^2 \left[q\left(\frac{b}{R}\right) + q\left(\frac{x}{R}\right) - \frac{\pi}{2}\right]\right\} & -R \leqslant x \leqslant 0 \\ \exp\left\{-\lambda R^2 q\left(\frac{x}{R}\right)\right\} & b \leqslant x \leqslant R. \end{cases} \tag{23}$$

Using these expressions, the expected progress of our packet is calculated as

$$Z(R) = p(1-p) \int_{-R}^{R} e^{-pn(x;R)}\, x\, dF(x;R) \tag{24}$$

or, in a normalized form,

$$\begin{aligned} Z(N;\beta)\sqrt{\lambda} &= 2p(1-p)\left(\frac{N}{\pi}\right)^{3/2} \Bigg\{ \int_r^1 t\sqrt{1-t^2} \\ &\cdot \exp\left\{-\frac{pN}{\pi}[q(r-t)+\pi-q(-t)] - \frac{N}{\pi}q(t)\right\} dt \\ &+ \int_{r-1}^{0} t\sqrt{1-t^2} \exp\left\{-\frac{pN}{\pi}[q(r-t)+\pi-q(-t)] \right. \\ &\left. - \frac{N}{\pi}\left[q(r)+q(t)-\frac{\pi}{2}\right]\right\} dt + \int_{-1}^{r-1} t\sqrt{1-t^2} \\ &\cdot \exp\left\{-\frac{pN}{\pi}[\pi - q(-t)] \right. \\ &\left. - \frac{N}{\pi}\left[q(r)+q(t)-\frac{\pi}{2}\right]\right\} dt \Bigg\} \end{aligned} \tag{25}$$

where

$$N = \lambda\pi R^2; \qquad r = b/R = \sqrt{\pi\beta/N}. \tag{26}$$

Fig. 11 shows the optimal radii N^* and R^* and the expected progress $Z(N; \beta)\sqrt{\lambda}$ for various values of gap intensity β. We see that, for fixed λ

$$N^* \cong \begin{cases} 7.7 & \text{as } \beta \to 0 \\ \infty & \text{as } \beta \to \infty \end{cases} \tag{27}$$

and

$$Z(N^*)\sqrt{\lambda} \cong \begin{cases} 0.043 & \text{as } \beta \to 0 \\ 0.050 \text{ (maximum)} & \text{at about } \beta = 1.0 \\ 0 & \text{as } \beta \to \infty. \end{cases} \tag{28}$$

The results for narrow gaps reduce to the case with homogeneous density. As the gap width increases, the expected progress increases because some of the possibly interfering terminals are removed by the gap. However, for too wide a gap, the transmission radius must be accordingly larger in order to cross it, which causes more conflicts at the receiver; thus, the expected progress decreases. It is interesting that the optimized expected progress achieves its maximum at about $\beta = 1$. We can also see that for fixed b

$$R^*/b \cong \begin{cases} \infty & \text{as } \beta \to 0 \\ 1 & \text{as } \beta \to \infty \end{cases} \tag{29}$$

and

$$Z(R^*)/b \cong \begin{cases} \infty & \text{as } \beta \to 0 \\ 0 & \text{as } \beta \to \infty. \end{cases} \tag{30}$$

Therefore, for large λ, the optimal transmission radius is just large enough to reach the other bank. However, the higher possibility of interference with the terminals behind the receiver (i.e., those in the area $x > b$) diminishes the value of $Z(R^*)/b$. For small λ, since there is almost no inter-

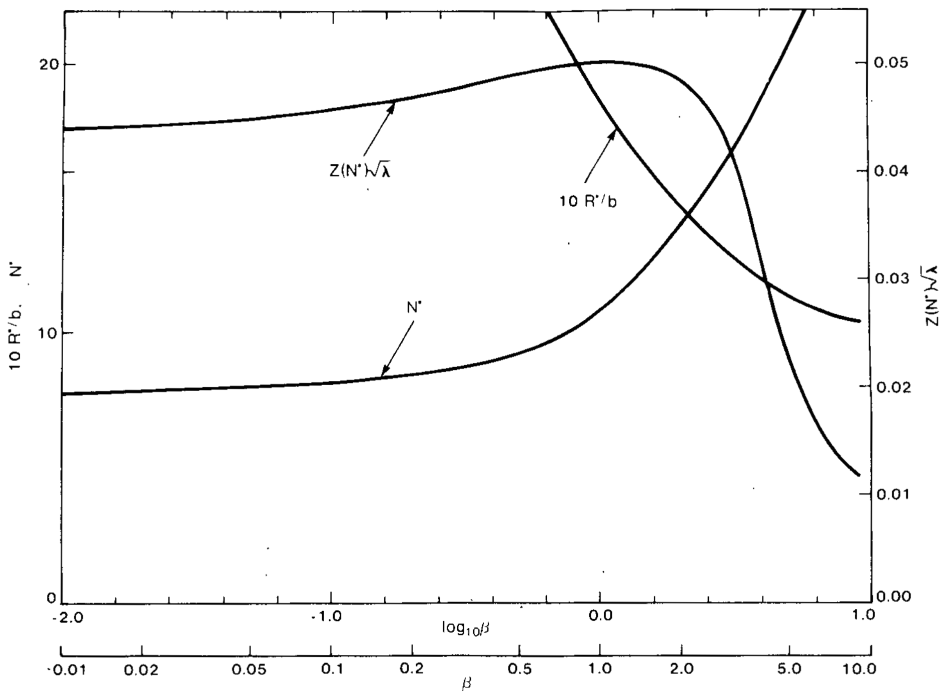

Fig. 11. The optimal transmission radii and expected progress for packet crossing a gap.

ference, the packet can proceed as far as an arbitrarily large transmission radius.

From Fig. 11, we see that the existence of the gap such that $\beta < 2$ helps the transmission. Notice, however, that a gap has an effect on performance only when $\beta \gg 1$. Thus, a conclusion here is that to cross the gap, we should not use a large transmission power with the same channel; rather, we had better use a separate channel (or wire) to avoid possible collisions.

VI. Optimal Number of Neighbors for ALOHA with Capture

In this section, we extend the model of a partially connected packet radio network with capture proposed by Fratta and Sant [2] to the context of our optimization problem. The reason for doing this is that the packet routing algorithm possibly implemented in each terminal is more suitably handled with their model than with the aforementioned MFR which assumes each terminal knows the position of an indefinite number of terminals within a distance R. However, without a notion of transmission radius, their model has a drawback of having an unrealistically wide area of interfering terminals for the case of poor capture (large capture ratio). Therefore, the results obtained here should be applicable only to the case of good capture.

The present model assumes a slotted ALOHA transmission protocol with transmission probability p in each slot, a Poisson distribution of terminals with homogeneous density λ, and an isotropic distribution of source–destination pairs. The concept of capture is described in Section II with capture parameter α. Every terminal is assumed to use the same transmission power. We do not use the notion of transmission radius, which implies that a transmission over a distance r is successful if none of the other terminals within the distance αr of the receiver transmit in the same slot.

We now explain the routing strategy employed here. Each terminal is assumed to know all the positions of its N nearest neighbors. Given a packet and its final destination, a terminal transmits to the most forward terminal in the direction of the final destination among those N neighbors whose positions are known. In case no terminals exist ahead, it transmits to the least backward neighbor. We call this routing MFN (most forward within N). MFN assumes that each terminal keeps the positions of only a fixed number of terminals, lending itself to easy implementation.

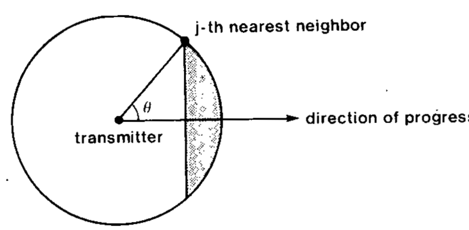

Fig. 12. The angular position of the jth nearest neighbor.

The routing algorithm at each terminal is formally stated as follows:

$j \leftarrow N$.

L: Consider j nearest neighbors. If the jth nearest one is the most forward, transmit to it.

Otherwise, $j \leftarrow j - 1$ and go to L.

This algorithm always terminates in at most N cycles. We can evaluate the routing probability $a_j(N)$ that the jth nearest neighbor is chosen as the receiver when considering N neighbors. To this end, let θ be the angular position of the jth nearest neighbor measured from the direction of progress, as shown in Fig. 12. Since the jth nearest neighbor is selected as the receiver only when it is the most forward among j neighbors, its probability is given by

$$\left(1 - \frac{1}{\pi} \left| \theta - \sin\theta \cos\theta \right| \right)^{j-1} \qquad -\pi \leqslant \theta \leqslant \pi. \tag{31}$$

(Notice that the distribution of the positions of up to the

TABLE I
THE ROUTING PROBABILITIES $a_j(N)$

j	$N = 1$	2	3	4	5	6	7	8	9	10
1	1.0000	0.5000	0.3017	0.1976	0.1359	0.0966	0.0704	0.0523	0.0395	0.0302
2		0.5000	0.3017	0.1976	0.1359	0.0966	0.0704	0.0523	0.0395	0.0302
3			0.3967	0.2598	0.1787	0.1271	0.0926	0.0688	0.0519	0.0397
4				0.3450	0.2373	0.1687	0.1230	0.0913	0.0689	0.0527
5					0.3122	0.2220	0.1618	0.1202	0.0907	0.0693
6						0.2890	0.2106	0.1564	0.1180	0.0902
7							0.2712	0.2015	0.1520	0.1162
8								0.2571	0.1940	0.1483
9									0.2455	0.1876
10										0.2356

$(j-1)$st nearest neighbors is no longer Poisson but uniform since we have specified j.) Unconditioning on θ with the isotropic assumption gives

$$c_j \triangleq \text{Prob}\,[\text{a terminal transmits to the } j\text{th nearest neighbor among } j \text{ of them}] = \frac{1}{\pi}\int_0^{\pi}\left[1-\frac{1}{\pi}(\theta-\sin\theta\cos\theta)\right]^{j-1} d\theta \qquad j=1,2,\cdots,N. \tag{32}$$

Using the above definition of c_j's, we may finally write the routing probability as

$$a_j(N) = c_j \prod_{k=j+1}^{N}(1-c_k) \qquad j=1,2,\cdots,N. \tag{33}$$

Clearly

$$\sum_{j=1}^{N} a_j(N) = 1. \tag{34}$$

In Table I, we show some values of $a_j(N)$.

From this point, we follow the derivation in [2]. First, the probability density function of the distance r_j to the jth nearest neighbor is given by

$$P_{r_j}(r) = \frac{2(\lambda\pi r^2)^{j-1}}{r(j-1)!}e^{-\lambda\pi r^2} \qquad r>0 \tag{35}$$

It follows that the mean distance to the jth nearest neighbor is

$$E[r_j] = \frac{(2j-1)!!}{2\sqrt{\lambda}(2j-2)!!} \tag{36}$$

where $(2j-1)!! = (2j-1)\cdot(2j-3)\cdots 3\cdot 1$ and $(2j)!! = (2j)\cdot(2j-2)\cdots 4\cdot 2$. Particularly, the mean distance between the two nearest neighbors is given by

$$E[r_1] = 1/(2\sqrt{\lambda}) \tag{37}$$

which (without the factor 1/2) we have used extensively to normalize the expected progress in the preceding sections.

Next, let S_j be the event that a packet transmitted to the jth nearest neighbor is successfully received. As shown in [2],

$$\text{Prob}\,[S_j \mid r_j = r] = (1-p)(1-pq)^{j-1}e^{-\lambda p\pi r^2(\alpha^2-q)} \tag{38}$$

where

$$q \triangleq \begin{cases} \dfrac{2}{3}-\dfrac{\sqrt{3}}{2\pi} \doteq 0.391 & \alpha = 1 \\ \dfrac{1}{\pi}\left\{\pi+(\alpha^2-2)\cos^{-1}\left(\dfrac{\alpha}{2}\right)-\alpha\sqrt{1-\frac{1}{4}\alpha^2}\right\} & 1 \leqslant \alpha \leqslant 2 \\ 1 & \alpha \geqslant 2. \end{cases} \tag{39}$$

Unconditioning on r_j with (35) gives

$$\text{Prob}\,[S_j] = \frac{(1-p)(1-pq)^{j-1}}{(1+\alpha^2 p - pq)^j}. \tag{40}$$

It follows that the throughput of transmission is given by

$$S(p,N;\alpha) = p\sum_{j=1}^{N} a_j(N)\,\text{Prob}\,[S_j]. \tag{41}$$

This completes our quotation from [2]. Similarly, we obtain the expected progress as

$$Z(p,N;\alpha) = \frac{p}{\sqrt{1+\alpha^2 p - pq}}\sum_{j=1}^{N} b_j(N)E[r_j]\,\text{Prob}\,[S_j] \tag{42}$$

where

$$b_j(N) \triangleq \prod_{k=j+1}^{N}(1-c_k)\cdot\frac{1}{\pi}\int_0^{\pi}\cos\theta \cdot\left[1-\frac{1}{\pi}(\theta-\sin\theta\cos\theta)\right]^{j-1} d\theta \qquad j=1,2,\cdots,N. \tag{43}$$

Our optimization problem is to find the maximum of $Z(p, N; \alpha)\sqrt{\lambda}$ in the (p, N) plane. In Fig. 13, the optimal values of N, N^*, and the maximum values of $Z(p, N; \alpha)\sqrt{\lambda}$, $Z^*\sqrt{\lambda}$, are plotted for various values of capture parameter α. It can be seen that the expected progress decreases rapidly as α increases, which does not agree with the result shown in Fig. 6. This comes from the present assumption that there is no fixed transmission radius. However, for small α, the results that $N^* = 7$ and $Z^*\sqrt{\lambda} \cong 0.05$ agree with the previous results. Therefore, we may conclude that 7 is suitable for the number of known terminals when the MFN routing is adopted.

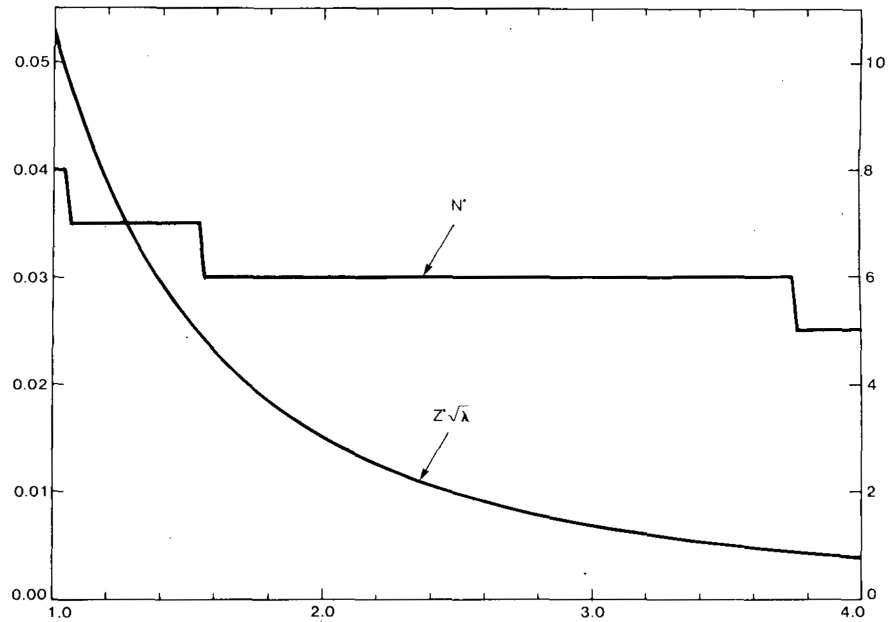

Fig. 13. The optimal transmissions for ALOHA with capture and without transmission radius.

VII. Conclusion

We have solved for the maximum expected progress per hop, (Z), provided by the optimal transmission probability (p) and transmission radius (expressed in terms of the number of terminals in the range, N), in some models of randomly distributed packet radio terminals (with average density λ) under the assumption of heavy traffic (all terminals always have ready packets). The quantity $Z\sqrt{\lambda}$ has been used consistently as the dimensionless objective function for optimization problems with respect to p and N. Major conclusions about the performance of each model are as follows.

The optimal transmission with slotted ALOHA without capture is attained by $N = 7.72$ and $p = 0.113$ which gives $Z\sqrt{\lambda} = 0.0431$. Therefore, each terminal transmits once in every nine slots on the average with the transmission radius covering just about eight nearest neighbors in the direction of packet's final destination. The probability of success of such a transmission is nearly equal to $1/e$. The expected progress per transmission is about two thirds of R/e, where R is the optimal transmission radius ($N = \lambda\pi R^2$).

FM capture improves the performance of slotted ALOHA systems due to the more limited area of possibly interfering terminals around the receiver. The expected progress in a system with perfect capture (optimized with $N = 7.1$ and $p = 0.17$) is about 36 percent greater than that in the system without capture. The probability of successful transmission is also higher than $1/e$. A model which is more amenable to implementation (each terminal knows the positions of only a fixed number of its neighbors) has shown similar results.

The slotted nonpersistent CSMA provides a nominal improvement in performance over the ALOHA system (16 percent improvement in the optimized expected progress for the zero propagation delay), which is not as large an improvement as we have obtained in the single-hop case. The reason for this is the large area of "hidden" terminals (about half of the interfering area) which cannot hear the transmission, and the long vulnerable period (twice as long as the packet transmission time) due to those terminals. The performance of (slotted nonpersistent) CSMA is comparable to that of ALOHA with good FM capture (capture ratio about 1.5 dB). The degradation occurs as the ratio of propagation delay to the transmission time increases.

As an example of an inhomogeneous terminal distribution, the effect of a gap of width b in an otherwise uniformly Poisson-distributed terminal population on the optimal transmission has been considered. The expected progress of a packet residing at the terminal on the bank and destined to cross the gap is evaluated with parameter $\beta = \lambda b^2$, called gap intensity. For fixed λ, the existence of the gap helps the progress for $\beta < 2$, because some of the possibly interfering terminals are removed by the gap. The maximum in the optimized expected progress occurs at about $\beta = 1$. Thus, to cross most gaps wider than the average interterminal distance, one had better not use a large transmission radius, but should more sensibly use a separate channel or wire.

Appendix

Determination of Transmission Rate for CSMA

In this Appendix, we derive the relation between transmission rate p' and channel-sensing rate p for slotted nonpersistent CSMA. See Section IV. As in [9], we assume that

$$p' = pP_I$$

where P_I is the probability that the channel is sensed idle. Since the probability of an empty slot is given by $e^{-p'N}$, the expected value of the idle period I (see Fig. 7) is

$$\bar{I} = \sum_{k=1}^{\infty} kae^{-kp'N}(1 - e^{-p'N}) = \frac{ae^{-p'N}}{1 - e^{-p'N}}.$$

On the other hand, the transmission period is $1 + a$. Therefore,

$$P_I = \frac{\bar{I}}{1 + a + \bar{I}} = \frac{ae^{-p'N}}{1 + a - e^{-p'N}}.$$

Thus, we have obtained an equation which determines p' in terms of p:

$$p' = \frac{ape^{-p'N}}{1 + a - e^{-p'N}}.$$

For $a \ll 1$, we have

$$p' = \frac{p}{1 + (p'/a)N}$$

which explicitly gives p'.

Using the optimal values in this case ($N = 5.3$ and $p'/a = 0.20$), we see that the actual transmission rate is 41 percent of the sensing rate.

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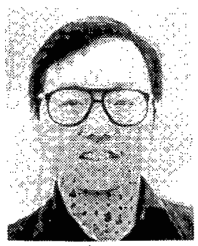

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Leonard Kleinrock (S'55–M'64–SM'71–F'73), for a photograph and biography, see p. 47 of the January 1984 issue of this TRANSACTIONS.